

Effects of strength anisotropy on the stability of slopes



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ABSTRACT

The effects of strength anisotropy of geomaterials in slope stability problems are the focus of this paper. The source of the strength anisotropy could be attributed to many factors and the motivation of this study is to concentrate on the effects of plane of weaknesses, such as joints, bedding planes etc. on the slope stability analysis. Two approaches are used in the finite element simulations to capture the effects of planes of weakness. In the first approach the planes of weakness are explicitly simulated by using joint networks, and in the second one a special constitutive model that considers embedded planes of weakness is utilized. Various slope stability problems have been analyzed using the shear strength reduction method to show the effects of anisotropy on the factor of safety of slopes.

RÉSUMÉ

Les effets de la résistance anisotropique des géomatériaux dans les problèmes de la stabilité des pentes sont l'objet du présent document. La source de la résistance anisotropique pourrait être attribuée à plusieurs facteurs. La motivation de cette étude est de se concentrer sur les effets du plan de faiblesse, telles que les joints, les plans de stratifications, etc. sur l'analyse de la stabilité des pentes. Deux approches sont utilisées dans les simulations des éléments finis pour saisir les effets des plans de faiblesses. Dans la première approche, ils sont explicitement simulés à l'aide des réseaux de joints discrets, et dans la deuxième approche un modèle spécial constitutive qui considère les plans de faiblesses intégrés est utilisé. Divers problèmes de stabilités des pentes ont été analysés en utilisant la méthode de résistance au cisaillement de réduction pour démontrer les effets d'anisotropie sur le facteur de sécurité des pentes.

1 INTRODUCTION

Natural soils and sedimentary rocks, such as shale, limestone and mudstone, are typically formed by deposition and progressive consolidation during formation. Such formations usually have a distinct internal structure, which is characterized by the appearance of multiple sedimentary layers. Besides the bedding planes, the geometric layout of networks of joints and other types of discontinuities in a rock mass are significant contributors to the complex behaviour of such geomaterials (e.g. Hoek and Brown 1980, Hoek 1983, Zienkiewicz and Pande 1977). The presence of these fissures and planes of weakness significantly influence the response of geotechnical structures such as slopes, tunnels and excavations (Goodman et al, 1968, Bandis et al, 1983).

2 MEDIA WITH ONE SET OF PLANES OF WEAKNESS

The influence of one set of planes of weakness on the behaviour of the geomaterial is investigated in this section. It is assumed that the weak planes are distributed uniformly in the medium. The spacing between these weak planes in the control volume of the material is small enough so that the combination of the matrix and weak planes can be smeared to an equivalent anisotropic

material. Such a configuration is illustrated in Figure 1 where the planes of weakness are oriented at an arbitrary angle α in the medium.

In this paper, it is assumed that the failure of the matrix and the planes of weakness can be described by Mohr-Coulomb and Coulomb criteria, presented in Equations 1 and 2 respectively.

$$(\sigma_1 - \sigma_3) - (\sigma_1 + \sigma_3) \sin \varphi - 2c \cos \varphi = 0 \quad [1]$$

$$\tau = \sigma_n \tan \varphi + c \quad [2]$$

In above φ and c are the friction angle and the cohesion, σ_1 and σ_3 are the major and minor principal stresses. τ and σ_n are the shear and normal stresses acting on the plane of failure. The sign convention is compression positive.

Two approaches are considered in the finite element simulations presented in this paper. In the first one the planes of weakness are simulated by using a closely spaced joint network in the domain of the problem. In the second one the planes of weakness are embedded in the constitutive model of the material. The constitutive model developed for commercial finite element package, Phase², considers Mohr-Coulomb criterion for the matrix

and can include up to three sets of weak planes for the material. Tension cut off criteria are also considered for the matrix and the weak planes. In the formulation of this constitutive model the total number of yield functions could add up to eight.

To illustrate the anisotropic effects induced by the planes of weakness a set of biaxial tests is considered in this section. The material properties for the matrix and the weak planes are listed in Table 1.

The variation of maximum axial stress with the inclination angle of the weak planes from both the analytical solution and the finite element simulations are presented in Figure 2. The finite element results are obtained by the use of the constitutive model that considers Mohr-Coulomb criterion for the matrix and Coulomb criterion for the weak planes, and the analytical solutions are from a simple evaluation of failure functions for the weak planes and the matrix for different configurations of the model under uniaxial loading (Pietruszczak 2010). Similar patterns for the variation of maximum axial stress with inclination angle has been reported by many researchers both in numerical and experimental investigations (e.g. Hoek and Brown 1980, Zienkiewicz and Pande 1977).

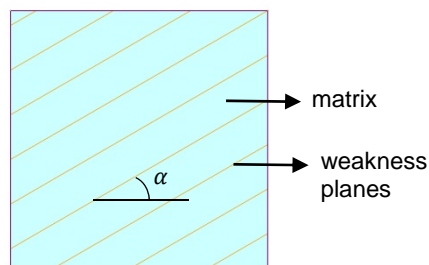


Figure 1. Typical control volume of a material with a set of weakness planes with inclination angle α .

Table 1. Properties for the matrix and weak planes.

Characteristics of the matrix	Value
Unit Weight	20 kN/m ³
Elastic Modulus	20 MPa
Poisson's Ratio	0.3
Friction Angle	37°
Cohesion	1000 kPa
Characteristics of the joint network	Value
Normal Stiffness	100 MPa
Tangential Stiffness	10 MPa
Friction Angle	20°
Cohesion	44 kPa
Characteristics of the plane of weakness	Value
Friction Angle	20°
Cohesion	44 kPa

Figure 3 presents the deformed shapes for different inclination angles. For the case of $\alpha = 0^\circ$ the sample fails due to failure of the matrix and the developed shear strain in the sample is zero. For the other two orientations the samples fail along the weak planes. Failure along the planes of weakness in these simulations results in non-zero shear strain in the sample.

An interesting point here is that for $\alpha = 30^\circ$ the distortion of sample complies with the direction of slip along the weak planes, but for $\alpha = 70^\circ$ the distortion of sample is in the opposite direction. Similar patterns of failure were observed when applying a set of joint network to simulate the weak planes (see Figure 4).

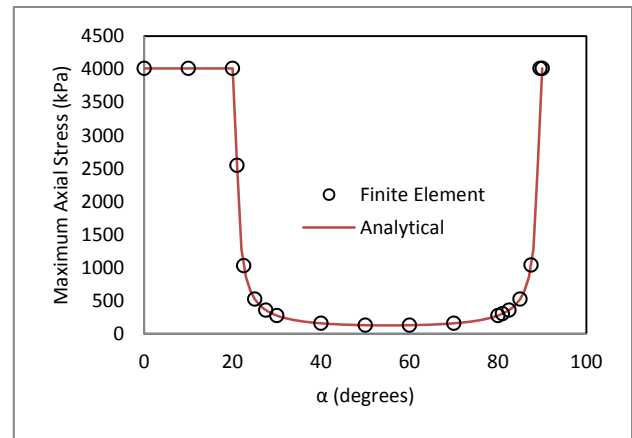


Figure 2. Variation of maximum axial stress of the material in biaxial test simulations with the inclination angle of the weak planes.

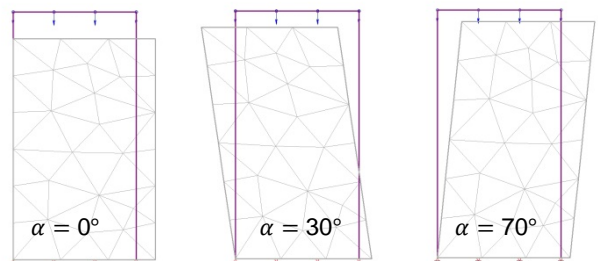


Figure 3. Original and deformed shape of biaxial samples for different inclination angles.

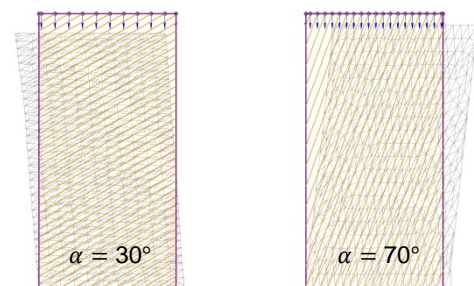


Figure 4. Original and deformed shape of biaxial samples with joint network inclination of $\alpha = 30^\circ$ and $\alpha = 70^\circ$.

2.1 Slope Stability Using SSR Simulations

The finite element method with the shear strength reduction factor (SSR) method (Hammah et al. 2006, Dawson et al. 1999, Griffiths and Lane 1999) is used to evaluate the safety factor of slopes against failure. In this method finite element analyses are used systematically to search for a strength reduction factor, i.e. the factor of safety, which brings a slope to the very limits of failure. Here, the strength reduction factor is applied to both the Mohr-Coulomb criterion for the matrix and the Coulomb criterion for the weak planes or joints by reducing the shear strength by the SSR factor.

The finite element simulations are carried out in two dimensional plane strain configuration using 6-node triangular elements. Figure 5 shows the geometry of the problem where α is the orientation of the planes with respect to the horizontal plane and $h=200\text{m}$. The external boundary of the model is constrained in both horizontal and vertical directions.

Two sets of slope stability simulations have been carried out in this paper.

In the first set, the effects of the orientation of the weak planes on the factor of safety are investigated. A range of inclination angle from 0 to 180 degrees is considered. The material properties are the same as the ones presented in Table 1. To have a better understanding of the effects of the weak planes on the stability of slopes, a separate simulation is carried out considering the same model with an isotropic material without any planes of weakness.

In the second set, to investigate the effect of relative strength of the weak planes with respect to the matrix, it is assumed that the weak planes are fixed and horizontal, i.e. $\alpha = 0^\circ$, and the shear strength of the weak planes including both the cohesion and the tangent of friction angle are factored to be 10 to 100% of the shear strength of the matrix.

Note that in these simulations the spacing between the weak planes is small enough so that the combination of the matrix and weak planes can be smeared to an equivalent anisotropic material. Individual faults or joints should be modeled by inserting actual joints in the finite element model.

3 NUMERICAL RESULTS AND DISCUSSIONS

Figure 6 presents the results of the first set of finite element simulations. The figures show distributions of maximum shear strain (distortion) in the domain of the problem obtained from finite element simulations utilizing the constitutive model with embedded weak planes. The results presented here are for the cases of no weakness planes and one set of weakness planes oriented at α equal to 0° , 25° , 45° , 65° and 135° , respectively.

The high intensity of distortion indicates the location of the slip surface. The values of safety factors are also included next to each model.

The first observation is that the shape of the failure surface is highly dependent on the orientation of weak planes. When there is no weak plane the slip surface is

circular (see Figure 6a). The presence of weak planes forces the failure surface to deviate from its original circular form. For $\alpha = 0^\circ$ and $\alpha = 25^\circ$ the noncircular failure surface and the influence of weak planes is obvious.

The evaluated safety factors are also highly affected by the inclination angle as they vary from 1 to about 3. Evidently the safety factor is higher in the absence of weak planes.

To better study the failure patterns predicted in finite element simulations, Figure 7 shows the locations and types of failure in the domain of the problem for the case of $\alpha = 0^\circ$.

In Figure 7a, corresponding to the simulation with embedded joints in the constitutive model of the material, the symbols "x" represents the failure of the matrix and symbol "+" is the failure along the weak planes. Clearly there is a concentration of "+" symbols at the toe of the slope where the horizontal slip surface was predicted (see Figure 6b). This concentration indicates that at this region the dominant mode of failure is along the weak planes. As the slip surface tilts up along the slope, the concentration of "x" symbols intensifies, meaning the failure takes place in the matrix. At some points where the "x" symbols and "+" symbols overlap, the material fails both in the matrix and on the weak planes.

The same pattern is observed in Figure 7b where the failure along the joints is illustrated by red lines and again the failure in the matrix is marked by "x" symbols. The pattern and distributions of the failures are in a good agreement in the two simulations.

For the first set of simulations the variation of safety factor with the orientation of the weak plans is presented in Table 2 and Figure 8. In all cases there is a good agreement between the results obtained from the two approaches used in simulations.

Table 3 and Figure 9 present the variation of safety factor with ratio of the strength of the weak planes over the strength of the matrix. The evaluated safety factor is lower in the presence of the weak planes where their strength is lower than the matrix. Once again there is a good agreement between the results produced by the two approaches applied to model the weak planes.

The time to complete the simulations that make use of the constitutive model with embedded joints is almost one tenth of the ones with joints network.

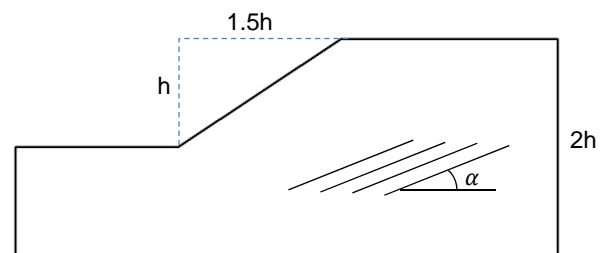


Figure 5. Geometry of the slope analyzed in this study.

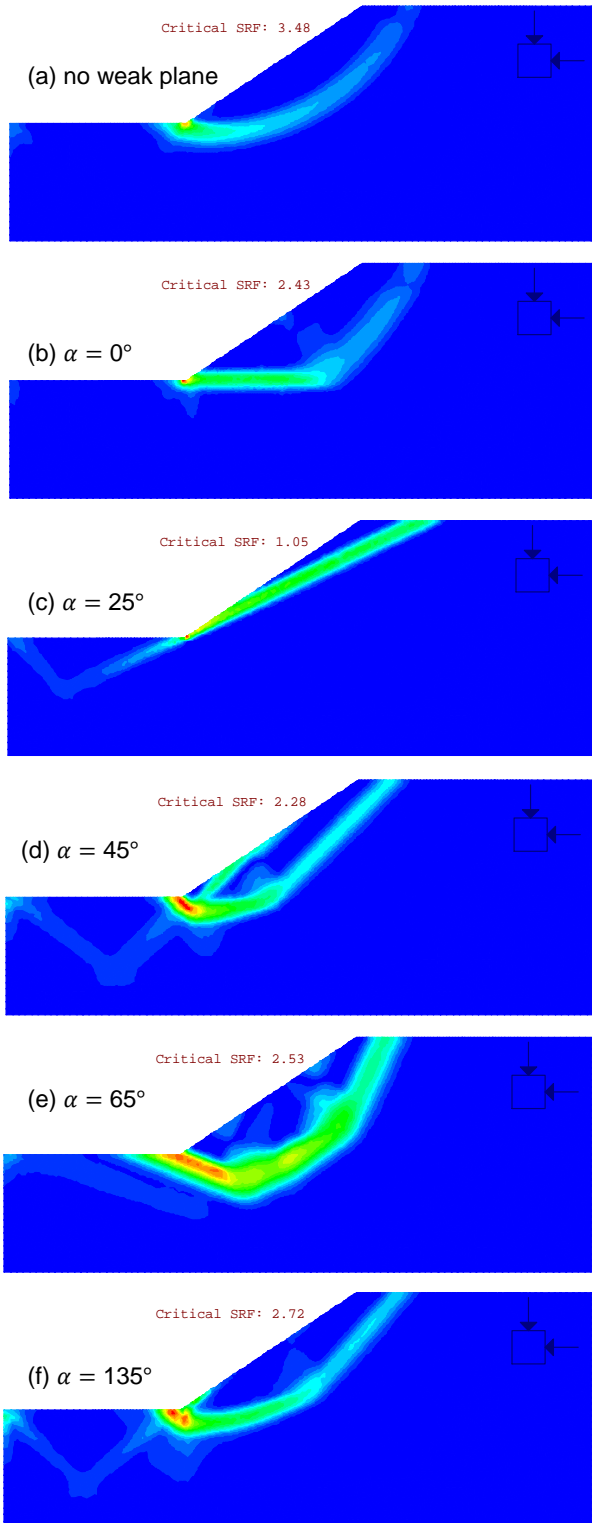


Figure 6. Numerical simulation results; the distribution of distortion in the domain obtained from finite element analyses using the constitutive model with embedded weak planes for the cases of no weakness planes and $\alpha = 0^\circ, 25^\circ, 45^\circ, 65^\circ$ and 135° .

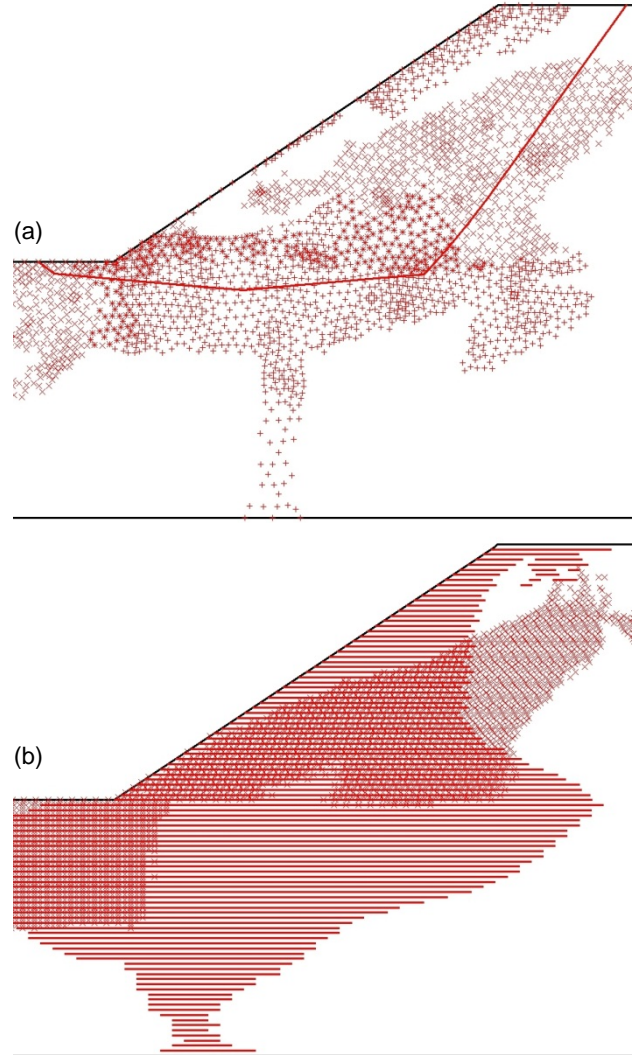


Figure 7. The predicted failure patterns for the case of $\alpha = 0^\circ$ in finite element simulations.

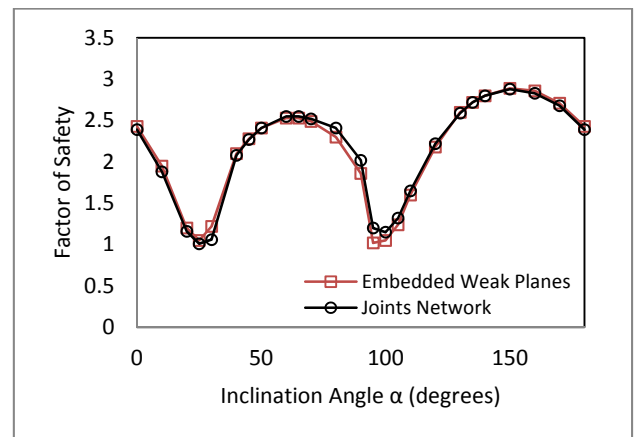


Figure 8. Variation of factor of safety with the inclination angle of weak planes.

Table 2. Evaluated Factor of Safety for different inclination angles of the planes of weakness.

α	SSR with Joint Network	SSR with Embedded Joints
None	3.48	3.48
0°	2.39	2.43
10°	1.88	1.95
20°	1.16	1.20
25°	1.01	1.05
30°	1.06	1.22
40°	2.08	2.10
45°	2.27	2.28
50°	2.41	2.41
60°	2.55	2.53
65°	2.55	2.53
70°	2.52	2.49
80°	2.41	2.30
90°	2.02	1.86
100°	1.15	1.05
110°	1.65	1.60
120°	2.22	2.18
130°	2.59	2.60
135°	2.72	2.72
140°	2.80	2.80
150°	2.88	2.89
160°	2.83	2.86
170°	2.68	2.71

weak planes were introduced to the model by inserting a joint network in the finite element model. In the second one the constitutive model of the material included the planes of weakness. It was shown that the stability of slopes is highly dependent on the presence and configuration of the weak planes in the material. The shape of the possible slip surface is also influenced by the orientation of the weak planes. The obtained numerical results using the two approaches were in a good agreement with each other in evaluation of the factor of safety of slopes.

Table 3. Evaluated Factor of Safety for different ratios of shear strength of weak planes compared to the strength of the matrix ($\alpha = 0^\circ$)

Strength ratio	SSR with Joint Network	SSR with Embedded Joints
1.0	3.44	3.48
0.9	3.38	3.43
0.8	3.27	3.34
0.7	3.15	3.23
0.6	3.02	3.08
0.5	2.85	2.90
0.4	2.65	2.69
0.3	2.41	2.45
0.2	2.11	2.17
0.1	1.76	1.82

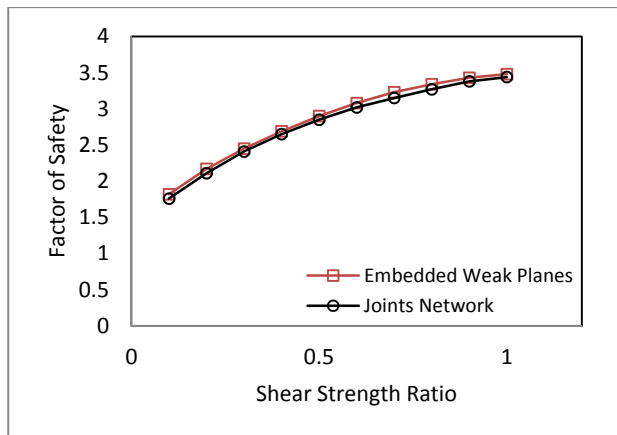


Figure 9. Variation of factor of safety with the ratio of shear strength of weak planes compared to the strength of the matrix ($\alpha = 0^\circ$)

4 CONCLUSIONS

The effects of strength anisotropy of geomaterials due to the presence of weak planes in the material on the stability of slopes were studied in this paper. The finite element method with shear strength reduction factor was used in the numerical analyses. Two different approaches were utilized in order to incorporate the effects of weak planes in finite element simulation. In the first one the

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