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Review

Poisson's ratio values for rocks

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Abstract

Compared to other basic mechanical properties of rocks, Poisson's ratio is an elastic constant of which the significance is generally underrated. Yet, in rock mechanics, there is a considerable number of diverse areas which require a prior knowledge or estimation of the value of Poisson's ratio. This paper examines the values and applications of Poisson's ratio in rock mechanics. Following an historical account of the initial controversy, whether it was a material constant or not, the effects of Poisson's ratio in the elastic deformation of materials, intact rocks, and rock masses are briefly reviewed. Also, the reported values of Poisson's ratio for some elements, materials, and minerals are compiled while typical ranges of values are presented for some rocks and granular soils. Finally, Poisson's ratio classifications are recommended for isotropic intact rocks.

Keywords: Poisson's ratio; Rocks; Rock masses; Deformability; Classification

Contents

1. Introduction

ISRM Commission on Terminology, Symbols and Graphic Representations defines Poisson's ratio as "the ratio of the shortening in the transverse direction to the elongation in the direction of applied force in a body under tension below the proportional limit" [1]. It is a surprising fact that this definition leaves much to be desired, i.e. it is mechanically inaccurate and unsatisfactory. To begin with, unless the initial dimension of the body parallel to loading is equal to its lateral dimension, the definition should involve strains not the dimensional changes such as shortening or elongation. Then, there is the question of missing negative sign before the ratio. Besides, the uniaxial loading may be not only tensile but compressive as well.

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Yet, the ISRM definition has not been corrected for about 30 years.

The importance of this mechanical property has not been appreciated as much as it deserves since the values of Poisson's ratio reported for rocks vary in a narrow range. Although the use of approximate or typical values in most rock mechanics applications does not create significant problems, Poisson's ratio plays an undeniably important role in the elastic deformation of rocks and rock masses subjected to static or dynamic stresses. Furthermore, its effects emerge in a wide variety of rock engineering applications, ranging from basic laboratory tests on intact rocks to field measurements for in situ stresses or deformability of rock masses. Therefore, information on various aspects of Poisson's ratio can be beneficial for rock engineering.

This paper aims to review the values of Poisson's ratio for rocks. First, some historical information on Poisson's ratio is summarized, and its importance in mechanics is emphasized. Then, its significance in rock mechanics is reviewed by particular references to minerals, intact rocks, jointed rock masses, and rock engineering applications. Also, recommendations are given for classification of intact rocks based on their Poisson's ratio.

2. Historical background

Thomas Young (1773–1829) drew the attention of his readers to a phenomenon in his *Course of Lectures* which was published in 1807. He noted that, during the experiments on tension and compression of bars, longitudinal deformations were always accompanied by some change in the lateral dimensions [2].

Siméon Denis Poisson (1781–1840), in his famous memoir [3], which was published in the year Young died but had been read to the Paris Academy on the 14th of April 1828 [4], made a proposal about an elastic constant that would create some controversy in the following years. For simple tension of an isotropic and elastic cylindrical bar with an original length of l and radius r, Poisson proposed that the radius had to become $r(1-0.25\delta)$ as the length became $l(1+\delta)$ by the deformation [2,4]. Based on an inadequate molecular model [5], this approach predicted the elastic constant, we now know as Poisson's ratio, to assume the value of 1/4.

The results of experiments later carried out by Guillame Wertheim (1815–1861), however, did not support Poisson's theoretical prediction [6]. Wertheim, using glass and metallic cylindrical tubes for the tests, measured changes of the internal volumes of tubes caused by the axial extension and, thus, calculated the lateral contraction [2,4]. Although the results could be explained by using two elastic constants for the isotropic materials, he continued to accept the so-called "uni-constant hypothesis," which assumed only one material constant (i.e. tensile or "stretch" modulus) for such materials. In 1848, Wertheim recommended the value of 1/3 be adopted for "the ratio of lateral contraction to longitudinal extension" without any theoretical basis or satisfactory agreement with the results of his experiments [2,4,6–8]. In his memoir of 1857, Wertheim also reported results of torsion experiments with prisms of circular, elliptical and rectangular cross sections or tubular specimens made of iron, glass and wood [7]. He concluded that "the stretch-squeeze ratio" was different from 1/4 and closer to 1/3 [2,4].

Similarly, tests carried out by A.T. Kupffer (1799–1865) on metal wires did not agree with the "uni-constant hypothesis," either. In 1853, Kupffer reported that the ratio of the modulus in tension to the "slide modulus" (i.e. shear modulus) determined from torsional vibration tests was different from 5/2, i.e. the value predicted by the hypothesis.

Franz Ernst Neumann (1798–1895), in his correspondence to Kupffer [7], assumed that the ratio of lateral contraction to longitudinal extension did not remain constant but depended upon the nature of material [2]. Kupffer also reported that Neumann, by fixing small mirrors to the sides of a rectangular bar under flexure, showed that its cross section became trapezoidal during bending [2,7]. By measuring the angle of relative rotation made by the two sides of the bar, Poisson's ratio could be calculated optically [2,7].

In 1859, Gustav Robert Kirchhoff (1824–1887), one of Neumann's pupils, attempted to settle the problem of uni-constancy by direct experiments he carried out on circular cantilever bars made of steel [8]. He applied a transverse load with a certain eccentricity to the free ends of cantilevers in such a way that bending and torsion were produced simultaneously. Then, the angle of torsion and the angle which the tangent at the end of the cantilever made with the horizontal were measured optically by using a mirror attached at the end of the cantilever [2]. Based on the results of experiments, Kirchhoff reported that "the stretch-squeeze ratio" was 0.297 for steel and 0.387 for brass, but he also expressed doubts about the absolute isotropy of the bars he used [8].

Barré de Saint-Venant (1797–1886), considering pure bending of a rectangular beam, established a basis for an experimental determination of Poisson's ratio. He showed that, when the beam was subjected to equal and opposite couples applied to the ends, initially rectangular cross section changed its shape as shown in Fig. 1 due to lateral contraction of the fibers on the convex side and expansion of those on the concave side. In fact, the initially straight line AB (i.e. neutral surface) becomes slightly curved upwards and corresponding radius of curvature is ρ/ν, where ν is Poisson's ratio and ρ is the radius of curvature of the axis of the bent bar (Fig. 1) [2]. Because of such a lateral deformation, the distances of the neutral fibers A and B from upper and lower surfaces of the bar are also slightly altered. Actually, all the surfaces parallel to neutral surface will be curved longitudinally downward and transversely upward, i.e. they are strained into anticlastic surfaces [6]. By determining the ratio of the two principal curvatures of

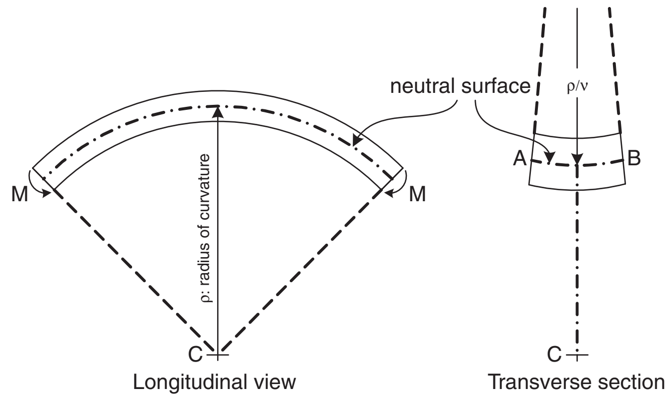


Fig. 1. Formation of anticlastic surfaces during uniform bending of an elastic beam with a rectangular cross section [2].

the anticlastic surfaces, Marie Alfred Cornu (1841–1902) carried out the first direct optical measurement of Poisson's ratio in 1869 [2,6,9]. In the experiments, he used glass bars and the value obtained was almost exactly 1/4 [6]. In 1879, H. R. Arnulph Mallock also reported about similar bending experiments for determination of Poisson's ratio for several materials [6].

Woldemar Voigt (1850–1919), another one of Neumann's pupils, between 1887 and 1889, carried out torsional and bending tests on thin prisms cut from single crystals in various directions and determined elastic moduli. The results definitely showed that, for isotropic elastic bodies, two material constants needed. In a sense, Voigt's work finally settled the controversy over uni-constant hypothesis [2].

Love [6] also reports about experiments for direct determination of Poisson's ratio by Pietro Cardani (1858–1925) and J. Morrow, both in 1903. Finally, in 1908, Eduard August Grüneisen (1877–1949) determined Poisson's ratio experimentally for the first time as the ratio of lateral and longitudinal strains in uniaxial tension tests [10]. This approach later became a basis for a common method of measurement of Poisson's ratio by static tests.

3. Poisson's ratio in mechanics

Before emphasizing the significance of Poisson's ratio in mechanics, an accurate definition of this interesting property should be made. There are numerous definitions of Poisson's ratio in the literature and many lack completeness. Poisson's ratio, simply, is the negative of the ratio of transverse strain to the axial strain in an elastic material subjected to a uniaxial stress. In mechanics of deformable bodies, the tendency of a material to expand or shrink in a direction perpendicular to a loading direction is known as the "Poisson effect."

To start with, Poisson's ratio is encountered in expressions involving Hooke's law. The value of this material property, which can be measured by static or dynamic methods, varies within a narrow range. Although the values of Poisson's ratio for many materials are close to the initial recommendation of 1/4 by Poisson or 1/3 by Wertheim, it is a well-known fact today that its theoretical value for an isotropic material is between −1 and 1/2 [2,4–6]. These lower and upper limits exist due to the fact that Young's (E), shear (G), and bulk (K) moduli of a material must be positive, based on thermodynamic restrictions [5,6,11]. As the value of Poisson's ratio approaches 0.5, as with the rubber like materials, the material easily undergoes shear deformations but resists volumetric deformation and becomes incompressible [12]. For such materials, shear modulus is much less than bulk modulus.

Although some sources [5,11] state that materials with negative Poisson's ratio are unknown, there are indeed examples of such materials. They include cellular solids such as polymer or metallic foams with inverted or re-entrant cell structure (e.g. $v \simeq -0.8$ for copper foam), anisotropic fibrous composites, and crystalline materials such as α-cristobalite [12–16]. Materials with negative Poisson's ratio demonstrate a "counterintuitive" behavior [13]: such solids laterally expand when stretched in one direction or vice versa. A solid with Poisson's ratio close to −1 would be the opposite of rubber (anti-rubber); it would be highly resistant to shear deformations but easy to deform volumetrically, i.e. shear modulus is much greater than bulk modulus [13–15]. Today, materials with a negative Poisson's ratio are called as "auxetic materials" or "auxetics" [17].

Poisson's ratios of some elements are listed in Table 1. Also, for some significant materials, values of Poisson's ratio are compiled in Table 2.

According to Tables 1 and 2, the values of Poisson's ratio for many elements and materials are between 0 and 0.5.

For isotropic and elastic solids, some quantities that depend only on Poisson's ratio can be expressed. The ratios of various elastic moduli are the primary examples of such quantities:

$$E/G = 2(1 + v), \tag{1}$$

$$E/K = 3(1 - 2v), \tag{2}$$

$$G/K = 1.5(1 - 2v)/(1 + v). \tag{3}$$

Also, the ratio of shear wave velocity (v_s) to the longitudinal wave velocity (v_p) in an isotropic solid with an infinite extent is another example:

$$v_s/v_p = [(0.5 - v_d)/(1 - v_d)]^{1/2}, \tag{4}$$

where v_d is the dynamic Poisson's ratio of the medium, and it can be different than that obtained from static tests. In addition, the ratio of Rayleigh wave velocity to the shear wave velocity ($\alpha = v_R/v_s$) depends only on the value of Poisson's ratio of the medium, and it can be found as the admissible (real and positive) root of the following equation [27]:

$$\alpha^6 - 8\alpha^4 + 8[(2 - v_d)/(1 - v_d)]\alpha^2 - 8/(1 - v_d) = 0. \tag{5}$$

Table 1
Poisson's ratio for some elements (data after Winter [18] and MaTecK GmbH [19])

Element		Poisson's ratio	Element		Poisson's ratio
Berillium	Be	0.032	Barium	Ba	0.28
Tellurium	Te	0.16–0.30	Praseodymium	Pr	0.281
Europium	Eu	0.152	Neodymium	Nd	0.281
Ytterbium	Yb	0.207	Magnesium	Mg	0.291
Chromium	Cr	0.21	Molybdenum	Mo	0.293
Plutonium	Pu	0.21	Caesium	Cs	0.295
Thulium	Tm	0.213	Cadmium	Cd	0.30
Uranium	U	0.23	Rubidium	Rb	0.30
Holmium	Ho	0.231	Calcium	Ca	0.31
Erbium	Er	0.237	Nickel	Ni	0.312
Manganese	Mn	0.24	Titanium	Ti	0.316
Dysprosium	Dy	0.247	Cobalt	Co	0.32
Cerium	Ce	0.248	Germanium	Ge	0.32
Zinc	Zn	0.249	Bismuth	Bi	0.33
Antimony	Sb	0.25–0.33	Sodium	Na	0.34
Osmium	Os	0.25	Tantalum	Ta	0.342
Ruthenium	Ru	0.25	Copper	Cu	0.343
Gadolinium	Gd	0.259	Aluminum	Al	0.345
Rhodium	Rh	0.26	Tin	Sn	0.357
Iridium	Ir	0.26	Lithium	Li	0.36
Rhenium	Re	0.26	Vanadium	V	0.365
Hafnium	Hf	0.26	Silver	Ag	0.367
Lutetium	Lu	0.261	Zirconium	Zr	0.38
Terbium	Tb	0.261	Platinum	Pt	0.39
Yttrium	Y	0.265	Palladium	Pd	0.39
Thorium	Th	0.27	Niobium	Nb	0.397
Iron	Fe	0.27	Gold	Au	0.42
Samarium	Sm	0.274	Silicon	Si	0.42
Scandium	Sc	0.279	Lead	Pb	0.44
Strontium	Sr	0.28	Selenium	Se	0.447
Tungsten	W	0.28	Thallium	Tl	0.45
Lanthanum	La	0.28	Indium	In	0.45
Promethium	Pm	0.28	Gallium	Ga	0.47

Table 2
Poisson's ratio for some materials

Material	Poisson's ratio	Source
Cork	~0	Lakes [12]
Diamond		
(natural)	0.10–0.29	Miyoshi [20]
(synthetic)	0.20	Miyoshi [20]
Concrete		
(28-day old)	0.10–0.21	Howatson et al. [21]
(high performance)	0.13–0.16	Persson [22]
Glass		
(quartz)	0.167	Bass [23]
(obsidian)	0.185	Bass [23]
(soda)	0.23	Howatson et al. [21]
(borosilicate)	0.25	Howatson et al. [21]
Sulfur	0.20–0.34	Bass [23]
Porcelain	0.208	Kumar et al. [24]
Tungsten carbide (WC)	0.222	Kumar et al. [24]
Cast iron		
(gray)	0.26	Howatson et al. [21]
(nodular)	0.28	Howatson et al. [21]
Steel		
(mild)	0.27–0.30	Howatson et al. [21]
(high strength)	0.30	Howatson et al. [21]
Shotcrete	0.25–0.29	Lorman [25]
Human dentine (dry)	0.29	Kinney et al. [26]
Perspex	0.311	Kumar et al. [24]
Ice (at 257 K)	0.324	Bass [23]
Aluminum 2024	0.33	Howatson et al. [21]
Brass (70 Cu/30 Zn)	0.35	Howatson et al. [21]
Lucite	0.358	Kumar et al. [24]
PVC (hard)	0.378	Kumar et al. [24]
Phosphor bronze (5% Sn)	0.38	Howatson et al. [21]
Epoxy resin	0.38–0.40	Howatson et al. [21]
Teflon	0.399	Kumar et al. [24]
Nylon	0.40	Kumar et al. [24]
Rubber	~0.50	Lakes [12]

In general, Poisson's ratio does not have an effect on the distribution of stresses in plane elasticity problems that do no involve body forces. Yet, for three-dimensional stress situations, the effect of Poisson's ratio can be striking. A typical example for such an effect is the formation of anticlastic surfaces in a rectangular beam subjected to uniform bending (Fig. 1). Moreover, Poisson's ratio influences the stresses resulting from bending of bars or plates, contact of elastic bodies, rotating discs, etc. [9].

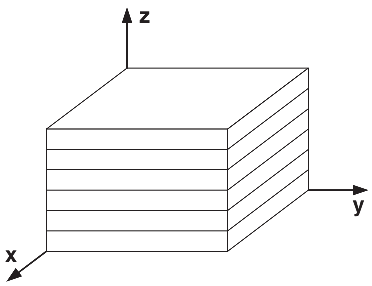
For elastic materials that demonstrate certain deformational anisotropy, multiple Poisson's ratios are expressed. For example, three Poisson's ratios are defined for transversely isotropic materials, and two of these are independent (Fig. 2). Although the elasticity theory does not impose certain limits on Poisson's ratios for such materials, there is a specific inequality derived from energy considerations. It is as follows [28,29]:

$$2\nu_2\nu_3 < 1 - \nu_1, \tag{6}$$

where ν_1, ν_2, and ν_3 are defined in Fig. 2. For Poisson's ratios defined in other directions, some unusual values may be obtained. In fact, it has been theoretically shown that Poisson's ratio for anisotropic materials can have an arbitrarily large positive or negative value as long as the strain energy density is positive definite [30,31]. Furthermore, for orthotropic elastic materials, six Poisson's ratios are defined (Fig. 3) and three of these are independent [27,28,32]. According to the notation in Fig. 3, ν_{ij} is Poisson's ratio defined by $-\varepsilon_j/\varepsilon_i$ for the uniaxial stress σ_i. Transverse isotropy and orthotropy are observed in certain rock types and jointed rock masses, and this issue is addressed in the next section.

4. Poisson's ratio in rock mechanics

Since Poisson's ratio is a mechanical property that plays a role in the deformation of elastic materials, it is utilized in rock engineering problems associated with the deformation of rocks, e.g. it is a required computational input for the numerical stress analyses. In the related literature [33,34], though very seldom, negative values or values greater than 0.5 are reported for Poisson's ratio of some rock types.

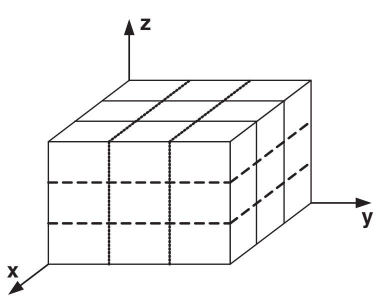


Non-zero stress	Elastic moduli	Poisson's ratios
σ_x	$E_1 = \sigma_x / \varepsilon_x$	$\nu_1 = -\varepsilon_y / \varepsilon_x$; * $\nu_3 = -\varepsilon_z / \varepsilon_x$
σ_y	$E_1 = \sigma_y / \varepsilon_y$	$\nu_1 = -\varepsilon_x / \varepsilon_y$; * $\nu_3 = -\varepsilon_z / \varepsilon_y$
σ_z	$E_2 = \sigma_z / \varepsilon_z$	$\nu_2 = -\varepsilon_x / \varepsilon_z = -\varepsilon_y / \varepsilon_z$
τ_{xy}	**$G_1 = \tau_{xy} / (2\, \varepsilon_{xy})$	--
τ_{yz}	$G_2 = \tau_{yz} / (2\, \varepsilon_{yz})$	--
τ_{zx}	$G_2 = \tau_{zx} / (2\, \varepsilon_{zx})$	--

*Dependent Poisson's ratio: $\nu_3 = (E_1/E_2)\, \nu_2$

**Dependent shearing modulus: $G_1 = E_1 / [2\,(1 + \nu_1)]$

Fig. 2. Definition of elastic moduli and Poisson's ratios for a transversely isotropic material with the plane of isotropy parallel to the x, y-plane.



Non-zero stress	Elastic moduli	Poisson's ratios
σ_x	$E_x = \sigma_x / \varepsilon_x$	$\nu_{xy} = -\varepsilon_y / \varepsilon_x$; * $\nu_{xz} = -\varepsilon_z / \varepsilon_x$
σ_y	$E_y = \sigma_y / \varepsilon_y$	$\nu_{yz} = -\varepsilon_z / \varepsilon_y$; * $\nu_{yx} = -\varepsilon_x / \varepsilon_y$
σ_z	$E_z = \sigma_z / \varepsilon_z$	$\nu_{zx} = -\varepsilon_x / \varepsilon_z$; * $\nu_{zy} = -\varepsilon_y / \varepsilon_z$
τ_{xy}	$G_{xy} = \tau_{xy} / (2\varepsilon_{xy})$	--
τ_{yz}	$G_{yz} = \tau_{yz} / (2\varepsilon_{yz})$	--
τ_{zx}	$G_{zx} = \tau_{zx} / (2\varepsilon_{zx})$	--

*$\nu_{xz} = (E_x/E_z)\, \nu_{zx}$; $\nu_{yx} = (E_y/E_x)\, \nu_{xy}$; $\nu_{zy} = (E_z/E_y)\, \nu_{yz}$

Fig. 3. Definition of elastic moduli and Poisson's ratios for an orthotropic material for which the x-, y-, and z-planes are the planes of symmetry.

Those few cases, probably, are associated with highly anisotropic rocks; also, it is reported that thermally induced microcracking in granites causes negative Poisson's ratio in compression and tension [35]. For isotropic rocks, therefore, the value of Poisson's ratio is practically between 0 and 0.5. In fact, the range bounded by the values of 0.05 and 0.45 covers most rocks. Also, in some rock engineering applications with limited field data, a value between 0.2 and 0.3 is a common estimate for Poisson's ratio.

4.1. Poisson's ratio of minerals

The values of Poisson's ratio for some minerals are listed in Table 3. It should be noted that the number of independent elastic constants appropriate to a mineral crystal depends on its crystal symmetry, and it ranges from three for a cubic crystal to twenty one for a triclinic crystal [23]. Therefore, for single crystals of minerals, it may not be possible to give a value for Poison's ratio. Using the values of elastic constants for anisotropic crystals, average isotropic elastic constants can be determined for a polycrystalline aggregate of the same material [36]. Bass [23] calculated and presented the Hill averages of the Voigt (upper) and Reuss (lower) bounds for isotropic bulk and shear moduli of some minerals. The Poisson's ratio values listed in Table 3 are calculated using the values of adiabatic bulk modulus and shear modulus for an equivalent isotropic polycrystalline aggregate.

An interesting item in Table 3 is α-cristobalite, a crystalline form of silica (SiO_2). Its Poisson's ratio varies between 0.08 and −0.5, depending on direction; in addition, the Voigt and Reuss bounds for Poisson's ratio of polycrystalline α-cristobalite are reported as $\nu_V = -0.13$ and $\nu_R = -0.19$, respectively [15]. Similarly, for a single crystal pyrite (FeS_2), Love [6] reported a negative Poisson's ratio ($\nu \simeq -1/7$) and suggested that this "somewhat paradoxical" value might be due to twinning of the crystals; yet, recent data ($\nu = 0.016$–0.160) [23] did not confirm this result.

4.2. Poisson's ratio of intact rocks

Although the values of Poisson's ratio for rock masses are required in majority of rock engineering applications, there are some instances when the values for intact rocks are necessary. For example, in overcoring methods employing the CSIR doorstopper, USBM borehole deformation gauge, CSIR triaxial strain cell, and CSIRO hollow inclusion cell, the value of Poisson's ratio for intact rock (i.e. stress relieved cores or overcores) is required for evaluation and interpretation of measurements [37,38]. In addition, the intact rock value can be considered as a limit for the values of Poisson's ratio that a jointed rock mass may assume. In Fig. 4, typical ranges of values are presented for Poisson's ratio of some rock types. It should be realized that some unusually extreme values are not included in the figure, and exceptions are always possible in the nature.

Generally, Poisson's ratio of intact rocks can be determined in the laboratory either indirectly by dynamic methods [39,40] or directly by static tests [41,42].

Table 3
Poisson's ratio for some minerals (calculated using data after Bass [23])

Mineral	Poisson's ratio
α-Cristobalite (SiO_2)	−0.164
Diamond (C)	0.069
α-Quartz (SiO_2)	0.079
Periclase (MgO)	0.182
Topaz ($Al_2(F, OH)_2\ SiO_4$)	0.221
Graphite (C)	0.223
Sapphire (Al_2O_3)	0.234
Magnesite ($MgCO_3$)	0.251
Halite (NaCl)	0.253
Magnetite (Fe_3O_4)	0.262
Galena (PbS)	0.270
Anhydrite ($CaSO_4$)	0.273
Rutile (TiO_2)	0.278
Chromite ($FeO \cdot Cr_2O_3$)	0.280
Albite ($NaAlSi_3O_8$)	0.285
Fluorite (CaF_2)	0.289
Dolomite ($CaMg(CO_3)_2$)	0.292
Calcite ($CaCO_3$)	0.309
Sphalerite (ZnS)	0.320
Uraninite (UO_2)	0.325
Gypsum ($CaSO_4 \cdot 2H_2O$)	0.336
Zincite (ZnO)	0.353
Bunsenite (NiO)	0.369
Celestite ($SrSO_4$)	0.379

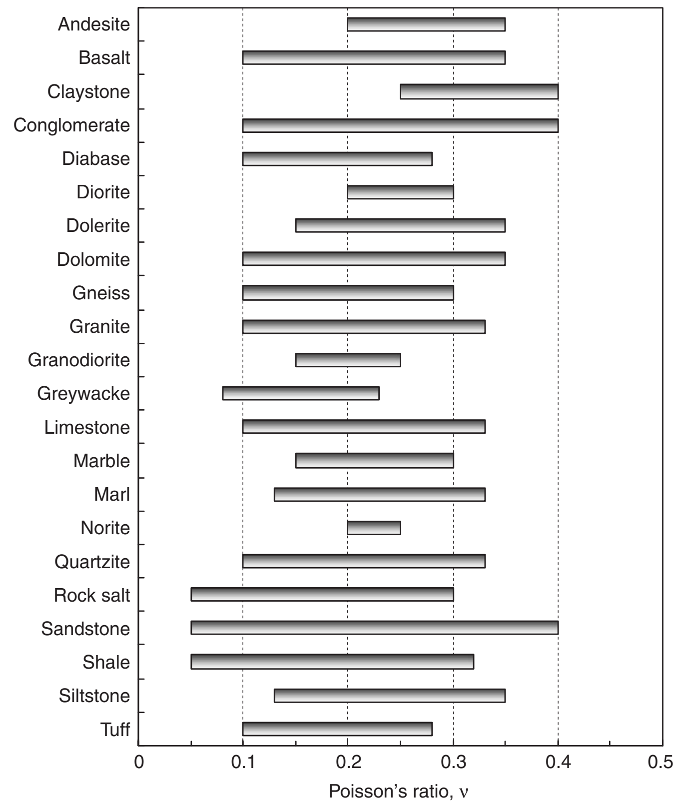


Fig. 4. Typical ranges of values for Poisson's ratio of some rock types (data after [33,34,44–46]).

The dynamic elastic tests involve either (i) determination of pulse velocities of longitudinal and shear waves in rock specimens or (ii) measurement of resonance frequencies of longitudinal and shear vibrations of bar or rod-like cylindrical rock specimens [39,40,43]. It has been reported that dynamic values of Poisson's ratio are often prone to considerable error [43].

In static tests by uniaxial compression for strength or deformability of rock material, it is recommended that the ratio of Young's modulus to Poisson's ratio (E/ν) of the platen material be close to that of the specimen to eliminate undesirable end effects [47]. For steel, a commonly used material as loading platen, the E/ν ratio is close to 670; yet, this value is generally larger than those of the rock types commonly encountered. Although aluminum ($E/\nu \simeq 200$) and brass ($E/\nu \simeq 300$) might provide a better match of E/ν ratio than steel, they can be easily damaged; for that reason, hardened steel platens with the same diameter as the test specimen are preferred [47].

Bieniawski, who studied the mechanism of brittle fracture of rock material in detail [48,49], proposed the criteria for identifying and separating the distinct phases of the failure process. According to Bieniawski, in cylindrical rock specimens under uniaxial compression, the variation of circumferential or radial strains with the axial stress starts to deviate from linearity at the transition from "linear elastic deformation" phase to that of "stable crack propagation." In other words, Poisson's ratio of the rock, which is constant during the linear elastic deformation, starts to increase due to initiation of new micro cracks or extension of existing ones [49]. Years later, similar results were obtained by more comprehensive studies [50]. Recently, Cai et al. [51] reported that, for a variety of rocks, the ratio of the crack initiation stress level to the uniaxial compressive strength fell in the range of 0.3 and 0.5 in uniaxial compression, and it varied between 0.36 and 0.6 in triaxial tests.

It has long been recognized that the nature of applied stress influences the mechanical properties of rocks. Values characterizing the uniaxial deformability (i.e. Young's modulus and Poisson's ratio) of rock material are expected to be different under compressive or tensile stress. The data reported by Krech et al. [52] definitely establish such a difference for Young's modulus in some rock types (e.g. granite, quartzite, sandstone, limestone, etc.). Similarly, the study by Liao et al. [53] on transversely isotropic argillite points out a somewhat less pronounced difference for Poisson's ratio. In addition, the values of elastic constants determined from static and dynamic tests differ due to some reasons (e.g. differences in applied stress or strain levels). Some studies [54–56] even suggested empirical relationships between static and dynamic Young's moduli of rocks. As far as Poisson's ratio of intact rock is concerned, there seems to be no conclusive evidence for such relationships. As a matter of fact, with the possible exception of porosity, there seems to be no meaningful correlation between the values of Poisson's ratio and any other mechanical or physical property of rock material [57,58]. Although one may anticipate that the porosity of rock material will play a role on the value of Poisson's ratio; however, the geometry (size and shape), orientation, distribution, and connectivity of pores are expected to complicate the influence. In this regard, the reader is referred to Walsh [59], who studied the influence of microstructure, especially porosity, on rock deformation. Also, in poroelasticity applications of geomechanics, the values of Poisson's ratio in drained and undrained conditions are required. It should be noted that undrained values of Poisson's ratio of rocks are larger than the drained values [60,61].

As mentioned earlier, transverse isotropy is a common feature of some sedimentary and metamorphic rocks with well-developed bedding planes. According to the deformability tests performed on intact rock materials, the values of v_1 and v_2 are always smaller than 0.5 while v_3 may assume values close to or larger than 0.5 [29]. In this context, the results of a study [62] involving static deformability of some coal measures are interesting and given in Table 4.

Finally, some important points to be considered regarding Poisson's ratio of coals may be summarized as follows.

(i) As the carbon content of coal exceeds 90%, its dynamic elastic constants, including Poisson's ratio, become increasingly anisotropic (i.e. transversely isotropic) [63].
(ii) A value of $v = 0.346$ was reported to be the representative Poisson's ratio for a wide range of coal grades [64].
(iii) According to a series of investigations [44–46], in which the static tests were carried out on laboratory specimens loaded perpendicular to bedding, Poisson's ratios of some Turkish coals were found to be between 0.15 and 0.49 (Table 5).

4.3. Poisson's ratio of rock masses

The behavior of rock masses are influenced by the mechanical behavior and properties of the discontinuities and those of the intact rock bounded by discontinuities. It has also been well known that structural features induce some degree of anisotropy in rock masses. For instance, transverse isotropy is observed in a rock mass with laminated fabric or one set of parallel discontinuities whereas orthotropic rock masses could arise when three mutually perpendicular sets of discontinuities with different properties and/or frequencies are present [32].

For the elastic deformability of jointed rock, joint normal stiffness (k_n), joint shear stiffness (k_s), and joint spacing are among the most important properties [65,66]. Amadei and Savage [65] and Amadei [66], who treated regularly jointed rock mass as an equivalent continuum with directional deformability properties that reflected the properties of intact rock and those of the joint sets, presented elastic stress–strain relations for transversely isotropic and orthotropic rock masses. They also gave expressions for apparent modulus and Poisson's ratios

Table 5
Poisson's ratios of some coals encountered at Zonguldak hardcoal region, Turkey [44–46]

Coal seam	Poisson's ratio	
(Colliery, sampling level)	Tangent[a]	Secant[a]
Acilik (Gelik, −150)	0.32	0.26
Acun (Gelik, −50/−150)	—	0.44
Akalin (Gelik, −150)	0.42	0.28
Buyuk (Kandilli, −300/−350)	0.34	0.28
(Kandilli, −450)	0.29	0.23
Cay (Gelik, −150)	0.28	0.28
(Asma, −170)	0.48	0.49
Hacimemis (Gelik, −260/−360)	0.33	0.26
(Asma, −170)	0.46	0.30
Kurul (Asma, −50)	0.16	0.15
Nasufoglu (Asma, −200/−250)	0.29	0.28
Ozkan (Gelik, −360)	0.32	0.24
Sulu (Gelik, −260/−300)	0.30	0.17
Taban Acilik (Asma, −50)	0.30–0.48	0.24–0.38
Tavan Acilik (Asma, 150;−250)	0.24–0.29	0.15–0.40

[a]Determined at 50% of uniaxial compressive strength.

Table 4
Poisson's ratios of some transversely isotropic rocks encountered at Zonguldak hardcoal region, Turkey (data after Colak [62])

Rock type (related coal seam, sampling position)	Poisson's ratios[a]		
	v_1	v_2	v_3
Sandstone (fine-medium grained) (Acenta, roof rock)	0.241	0.299	0.339
Sandstone (fine-medium grained) (Acilik, floor rock)	0.208	0.292	0.363
Sandstone (medium grained) (Buyuk, roof rock)	0.217	0.322	0.364
Sandstone (fine grained) (Cay, floor rock)	0.173	0.274	0.411
Sandstone (medium grained) (Cay, roof rock)	0.263	0.243	0.393
Sandstone (medium grained) (Domuzcu, floor rock II)	0.261	0.283	0.325
Claystone (Domuzcu, floor rock I)	0.281	0.363	0.482
Siltstone (Nasufoglu, roof rock)	0.231	0.287	0.515
Siltstone (Sulu, roof rock)	0.218	0.318	0.364

[a]Defined in Fig. 2.

of regularly jointed rock masses and showed that their values converged to those of an intact isotropic medium when the joint spacings or joint stiffnesses approached infinity.

There have also been some numerical studies to predict the value of Poisson's ratio for jointed rock masses [67–71]. In the majority of cases, the value of Poisson's ratio for the rock mass was larger than the value for the intact rock, and sometimes, unusually high values ($v > 0.5$) were obtained [67–70], indicating the anisotropy induced by the joints. In a recent study, Min and Jing [69,70] investigated the deformability of a two dimensional, randomly jointed rock mass model by a hybrid technique. In the study, the discrete fracture network approach was used to build up the model of fracture systems while a distinct element code was used for the numerical stress analysis. The analyses revealed that the value of Poisson's ratio for the rock mass was stress dependent and sensitive to the ratio of shear stiffness to normal stiffness of joints (k_s/k_n). In another study by Kulatilake et al. [71], rock fracture data belonging to a certain site were used to build up a three-dimensional stochastic fracture network model for a 30-m cube of jointed diorite, and a procedure involving a three-dimensional distinct element code was developed to estimate the strength and deformability of such a "virtual rock mass." The value of Poisson's ratio for the rock mass was found to be about 20% higher than the value for the intact rock.

Among the in situ tests to determine the deformability of rock mass, some require a prior knowledge or estimation of the value of Poisson's ratio while some others are used for direct determination of Poisson's ratio along with the rock mass modulus. In borehole expansion tests involving flexible dilatometers or stiff borehole jacks (e.g. Goodman jack), the value of Poisson's ratio for the rock surrounding the measurement section is assumed in order to determine the in situ deformation modulus. In the case of borehole jacking test, a specific coefficient that depends on the rock mass Poisson's ratio is used for calculation of the modulus of deformation [29,72].

For in situ determination of Poisson's ratio of a rock mass, several methods are available and, depending on the method, the volume of rock mass involved ranges from a fraction of a cubic meter to a significantly large volume. For this purpose, Lu [73] proposed a method that employed cylindrical and flat hydraulic borehole pressure cells developed by the USBM, and he presented results of measurements carried out in a coal seam. Detailed information is available [74] on the equipment, technique, and theories for the hydraulic borehole pressure cells that can also be used for determination of premining and mining-induced pressures and/or pressure changes.

Another possibility for in situ determination of rock mass Poisson's ratio is to use large flat jacks. Special procedures involving biaxial and triaxial flat jack tests were presented for determining the elastic constants of schistose rock masses with transverse isotropy [29].

Table 6
Typical ranges of values of Poisson's ratio for granular soils [81]

Soil type	Poisson's ratio
Loose sand	0.20–0.40
Medium dense sand	0.25–0.40
Dense sand	0.30–0.45
Silty sand	0.20–0.40
Sand and gravel	0.15–0.35
Saturated cohesive soils	~0.50

Boyle [75], who criticized the ISRM-suggested plate loading test [76] that required Poisson's ratio of the rock mass to be a known value, recommended an alternative numerical computation method to determine the value of Poisson's ratio along with the deformation modulus. Based on this recommendation, Unal [77], presented information on the test set-up, testing procedure, and derivation of formulas for a new approach developed for determining in situ deformability of rock masses.

Compared to the methods employing borehole pressure cells, large flat jacks, and plate loading; the dynamic in situ tests, in which the seismic velocities are measured, may be the only alternative for determining the value of Poisson's ratio for very large volumes of rock masses.

As it has also been shown in some studies [78–80], the rock mass deformation modulus (E_m) can be empirically correlated to the intact rock modulus (E_i). Unfortunately, there seems to be no such correlation between the values of Poisson's ratio for rock mass (v_m) and intact rock (v_i). Yet, theoretically, the intact rock (i.e. matrix) value constitutes a limit for the values that may be assumed by the jointed rock mass.

Finally, for the sake of completeness, typical ranges of values of Poisson's ratio for granular soils are given in Table 6.

4.4. Poisson's ratio in rock engineering

In rock mechanics and rock engineering, Poisson's ratio deserves a special consideration in many respects. In the preceding sections, its role and employment in some in situ tests are mentioned. Though not exhaustive, following are some additional areas in which the significance or influence of Poisson's ratio is appreciated.

Poisson's ratio of the medium influences the distribution of stresses in some three-dimensional solutions that are widely applied to geomechanics problems. Following are the important examples of such fundamental solutions [82,83]:

(i) point load acting in the interior of an infinite elastic body (Kelvin's problem),
(ii) point load acting normal to the surface of an elastic half space (Boussinesq's problem),
(iii) point load acting tangential to the surface of an elastic half space (Cerruti's problem), and

(iv) vertical or horizontal point load acting in the interior of an elastic half space with a horizontal surface (Mindlin's problems).

As one of many practical applications of the fundamental solutions, an integrated form of Boussinesq's problem is illustrated in Fig. 5. It shows the variation of stresses occurring along the centerline of a uniformly loaded circular area located on the surface of an elastic half space. The effect of Poisson's ratio is noticeable on the induced horizontal stresses (σ_h) while the induced vertical stress (σ_v) is independent of the elastic properties of the medium.

In rock engineering applications involving underground openings, Poisson's ratio of the rock mass is utilized for estimating in situ stresses and in expressions involving induced stresses. For example, in an approach that was attributed to Terzaghi and Richart [85] but had been used earlier by Mindlin [86], the ratio of horizontal in situ stress (P_h) to the vertical component (P_v) in geologically undisturbed sedimentary regions is as follows:

$$P_h/P_v = v/(1-v). \tag{7}$$

Similarly, in transversely isotropic rock masses with horizontal bedding planes, Eq. (7) becomes [29,37,87]:

$$P_h/P_v = [v_2/(1-v_1)](E_1/E_2), \tag{8}$$

where v_1, v_2, E_1, and E_2 are defined in Fig. 2. Amadei and Pan [87] also presented comparable expressions for orthotropic rock masses. In addition, Sheorey et al. [88] proposed the following expression for estimating P_h (in MPa):

$$P_h = [vP_v + \beta G_t E(H+1000)]/(1-v), \tag{9}$$

where P_v and E (Young's modulus) are in MPa, β (°C^{-1}) is the coefficient of linear thermal expansion, G_t (°C/m) is geothermal gradient, and H(m) is depth of cover. Obviously, for the estimation of P_h, there are other approaches [37] that do not involve Poisson's ratio of the rock mass. Also, Eqs. (7) to (9) are not valid when the assumptions made in their derivation are violated.

In the analytical solutions for stresses around underground openings, the following points are worth mentioning regarding the influence of Poisson's ratio.

(i) In plane-strain solutions that ignore body forces, only the axial (longitudinal) stress component (σ_z) involves Poisson's ratio:

$$\sigma_z = P_z + v[\sigma_\rho + \sigma_\theta - (P_v + P_h)], \tag{10}$$

where P_z is the principal in situ stress parallel to the longitudinal axis of opening, σ_ρ and σ_θ are radial and tangential stresses, respectively, occurring around the opening.

(ii) In the plane-strain solutions that include body forces, Poisson's ratio of the surrounding rock affects the stresses occurring around the opening. In this respect, the solution by Mindlin [86] for a horizontal circular tunnel located in a semi-infinite elastic solid under the action of gravity and Savin's solution [89] for distribution of stresses around a circular hole in an infinite heavy elastic plate are two relevant examples. In these solutions, the influence of Poisson's ratio is more pronounced on the opening boundary and at shallow depths. In Fig. 6, the variation of tangential stress occurring at the crown of a shallow circular tunnel is shown as a function of depth. It is obtained by using the generalized form of Mindlin's solution given by Gercek [90]. The results for a shallow tunnel are also compared with the Kirsch solution [91] for a circular tunnel located at great depth (Fig. 6).

(iii) Solutions involving three-dimensional (e.g. spherical, spheroidal, or ellipsoidal) openings include Poisson's ratio. For example, the expression for the radial stress occurring at a point around a spherical opening subjected to vertical in situ stress (P_v) is a striking example, and it is as follows [92]:

$$\sigma_\rho = P_v\{[14 - 10v - (38-10v)(a/r)^3 + 24(a/r)^5] + [10v - 14 + (50-10v)(a/r)^3 - 36(a/r)^5]\sin^2\theta\}/(14-10v), \tag{11}$$

where a is the radius of spherical opening, r is the distance to the center of opening, and θ is the angle between radial and vertical directions.

Some geomechanics problems with no mathematical solutions are commonly studied by numerical stress

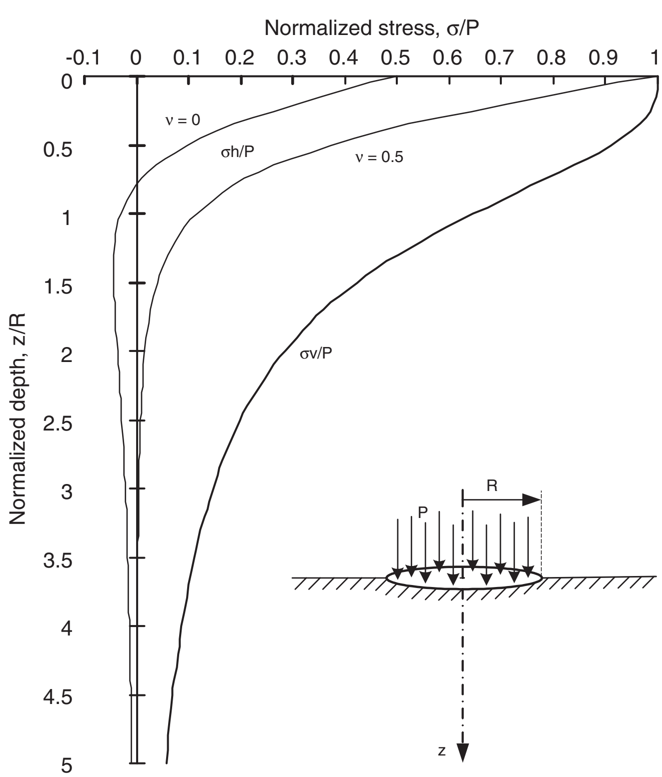


Fig. 5. Variation of the vertical (σ_v) and horizontal (σ_h) stresses occurring along the centerline of a uniformly loaded circular area located on the surface of an elastic half space (modified after [84]).

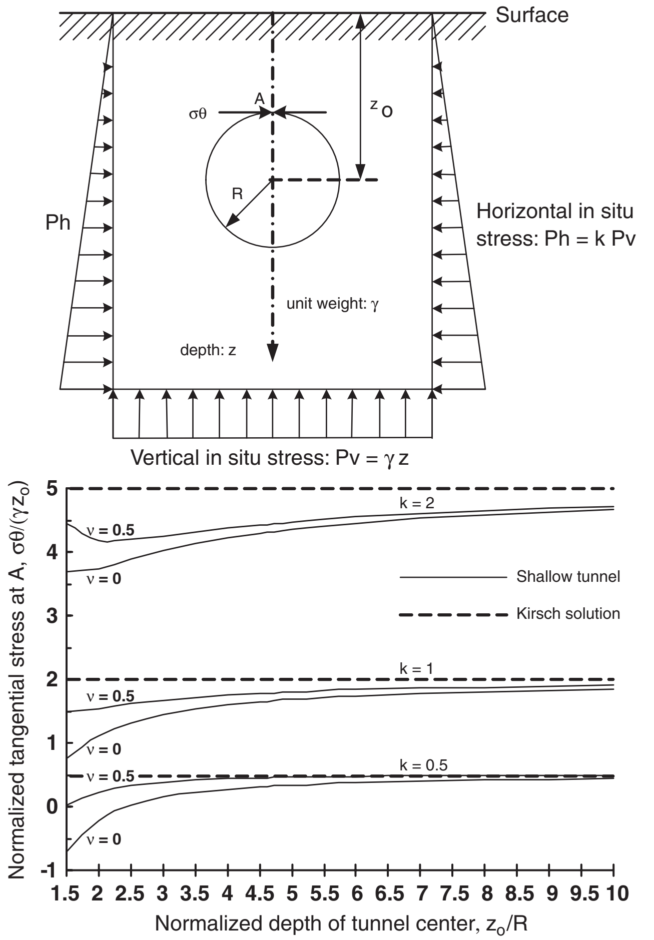


Fig. 6. Effect of Poisson's ratio of the medium on the tangential stresses occurring at the crown of a shallow circular tunnel.

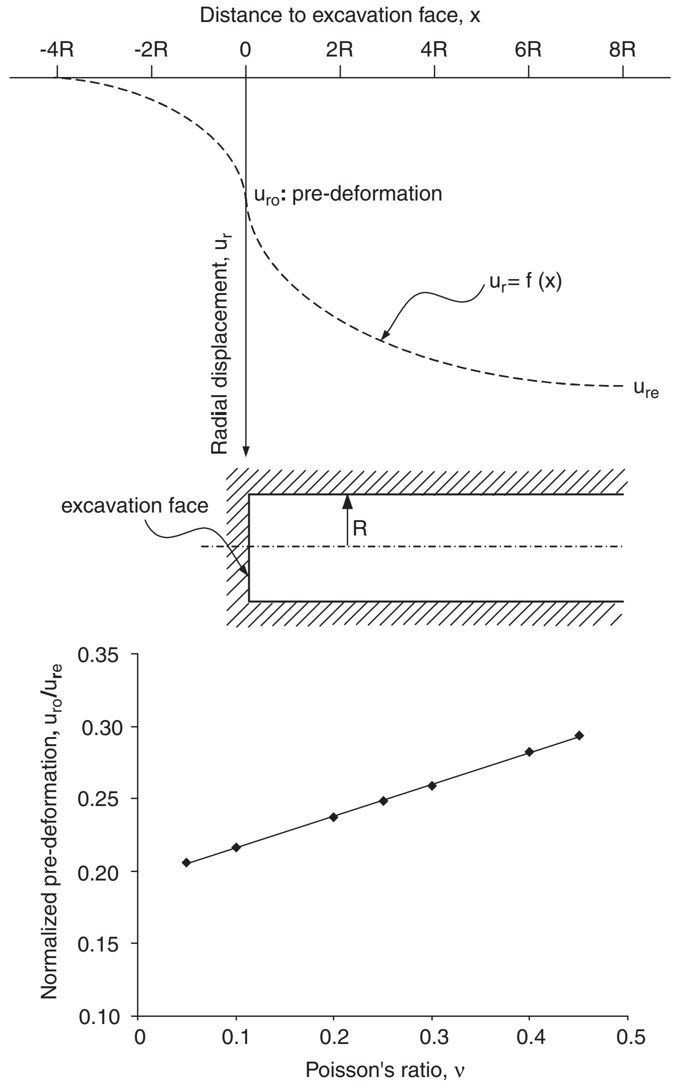


Fig. 7. Variation of the normalized elastic pre-deformation with the value of Poisson's ratio for a circular tunnel located in a hydrostatic in situ stress field [93].

analysis. The distribution of stresses and displacements occurring around the advancing face of a tunnel is an interesting example of such problems. It has been shown by a numerical study that Poisson's ratio of the surrounding medium influences the normalized elastic radial displacements occurring around the excavation face of a circular tunnel located in a hydrostatic in situ stress field [93]. This influence is illustrated in Fig. 7 for the normalized elastic pre-deformation, the ratio of radial displacement occurring at the face to that occurs far behind the face (i.e. u_{ro}/u_{re}). In a similar problem, the influence of Poisson's ratio was noted on the stress concentrations around the bottom of a borehole and core-disking phenomenon [94].

5. Recommendations for classification

During the preparation of this review, it was noticed that there was not any Poisson's ratio classification for rocks although a number of classifications existed about some mechanical, physical and index properties of intact rocks. For example; those involving the uniaxial compressive strength (σ_{ci}) [95–101], Young's modulus (E) [99], cohesion (c) [102], unit weight (γ) [103], point load strength index ($I_{s(50)}$) [104,105], slake durability index (I_d) [106,107], block punch index (BPI) [108], modulus ratio (E/σ_{ci}) [96,109], and point load strength anisotropy index ($I_{a(50)}$) [110] show the diversity of classifications for intact rock. Poisson's ratio is no less significant than some of the intact rock properties for which classifications have been proposed. In fact, a Poisson's ratio classification can be useful for a qualitative assessment of laboratory test result.

For classification of intact rocks based on their Poisson's ratio, two practical alternatives may be considered since the theoretical upper limit is 0.5 and there seems to be an observed lower limit of zero. In the first alternative with five categories (i.e. very low, low, medium, high, and very high), it is suggested that a range of 0.1 be chosen for each category (Table 7). In the second one with three categories (i.e. low, medium, and high), a range of 1/6 is recommended for each category (Table 8). It should be noted that these classifications are applicable to isotropic rocks only,

Table 7
Recommendation for a Poisson's ratio classification with five categories

Category	Poisson's ratio
Very low	$0 \leqslant \nu < 0.1$
Low	$0.1 \leqslant \nu < 0.2$
Medium	$0.2 \leqslant \nu < 0.3$
High	$0.3 \leqslant \nu < 0.4$
Very high	$0.4 \leqslant \nu < 0.5$

Table 8
Recommendation for a Poisson's ratio classification with three categories

Category	Poisson's ratio
Low	$0 \leqslant \nu < 1/6$
Medium	$1/6 \leqslant \nu < 1/3$
High	$1/3 \leqslant \nu < 1/2$

and if, by any chance, a negative value of Poisson's ratio is measured for a rock type, it has to be called "auxetic rock" in either of the classifications.

The merit of considering equal ranges for the categories may be questioned since the categories chosen for classifications involving uniaxial compressive strength, Young's modulus, etc. do not have equal ranges. However, it should be recalled that, unlike Poisson's ratio, those mechanical properties do not have a definite upper limit. Also, when there is not any upper limit, it has been customary to divide the possible range nonlinearly. Actually, in this paper, the main reason for recommending categories with equal ranges is that it is simple and easy to remember. Finally, it is hoped that these recommendations do not create a situation similar to the inconsistency that exists among the classifications for uniaxial compressive strength [111].

6. Conclusions

Poisson's ratio is an interesting mechanical property of elastic solids. Its significance in mechanics and rock engineering applications is much greater than that is implied by the narrow range of values it usually assumes. The data compiled in the paper can be used in engineering applications which require an estimation of Poisson's ratio. Also, the classifications recommended for Poisson's ratio of rocks are simple and easy to remember, and they can be utilized for qualitative grouping of quantitative test data.

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